Infinite duration two player zero sum turn based
perfect information deterministic qualitative games
& stochastic & quantitative

**Graphs**

\[ G = (V, E) \]
\[ E \subseteq V \times V \]

No sinks:
\[ \forall v \in V \exists (v, v') \in E \]
Players

Arenas

\[ V = V_{Eve} \cup V_{Adam} \]

\[ A = (V = V_{Eve} \cup V_{Adam}, E) \]

Convention: Eve ○
Adam □

Play

\[ T = (v_0, v_1, v_2, \ldots) \]

\[ \text{Play} = \text{Play}_{Eve} \cup \text{Play}_{Adam} \]

\[ \text{Play}_{Eve} = \{ (v_0, v_1, \ldots, v_k) : v_k \in V_{Eve} \} \]

\[ \text{Play}_{Adam} = \{ v \in V_{Adam} \} \]
Objectives
always: for Eve

$$\Omega \subseteq C^w \quad C \text{ colours}$$

Games

$$G = (A, \Omega [\text{col}])$$

$$\text{col} : V \rightarrow C$$

Example:

$$\Omega = \{ \pi : \exists i \in N, \pi_i = b \}$$
Strategies
1: Play_{Eve} \rightarrow E
2: Play_{Adam} \rightarrow E

Implicitly:
\begin{align*}
&\sigma \sim_{Eve} \bigcirc \\
&\sigma \sim_{Adam} \square
\end{align*}

Plays consistent with a strategy:

\[ \Pi = (v_1, v_2, \ldots) \text{ is consistent with } \sigma \]

\[ \forall i \in \mathbb{N} \quad v_i \in V_{Eve} \Rightarrow \sigma(v_1, \ldots, v_i) = (v_i, v_{i+1}) \in E \]

Similar: \( \Pi \) consistent with 2

Winning strategies
\( \sigma \) is winning from \( v \) if \( \forall \Pi \) consistent with \( \sigma \) starting in \( v \)

\[ \text{col}(\Pi) \in \mathcal{N} \]

Similar: 2 winning from \( v \)
Winning regions

\[ W_{\text{Eve}}(G) = \{ v \in V : \exists \sigma \text{ winning from } v \} \]

\[ W_{\text{Adam}}(G) = \{ v \in V : \exists \tau \text{ winning from } v \} \]

Determinacy: we say that G is determined if:

\[ W_{\text{Eve}}(G) \cup W_{\text{Adam}}(G) = V \]

\[ [W_{\text{Eve}}(G) \cap W_{\text{Adam}}(G) = \emptyset \text{ is trivial and always holds}] \]
Solving a game:

**INPUT:** \( G \) a game \( v_0 \in V \)

**OUTPUT:** Does Eve have a winning strategy from \( v_0 \)?

\( v_0 \in W_{Eve}(G) \) ?

\[
\begin{align*}
\text{[m]} & \quad \text{number of vertices} \\
\text{[m]} & \quad \text{number of edges}
\end{align*}
\]