Infinite duration two player zero sum turn based
≠ concurrent
perfect information deterministic qualitative games
≠ stochastic ≠ quantitative

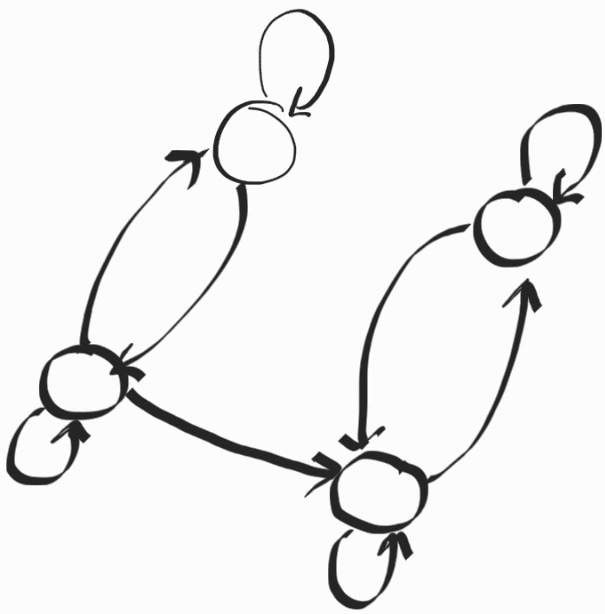
Graphs

$G = (V, E)$

$E \subseteq V \times V$

NO SINKS :

$\forall v \in V \ \exists (v, v') \in E$

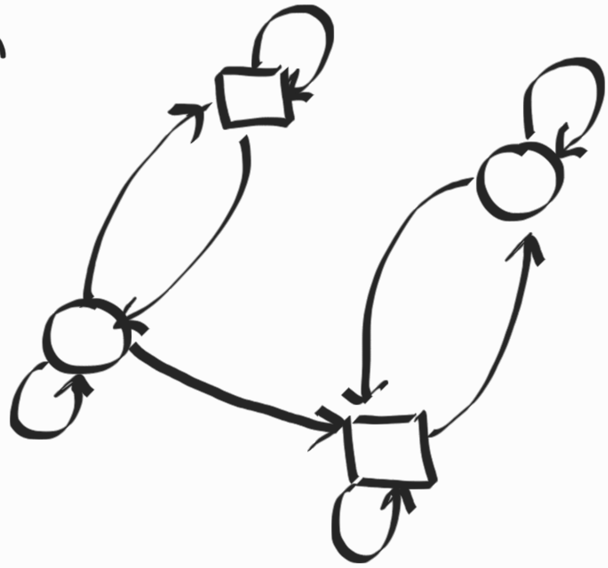


Players

Arenas

$$V = V_{Eve} \uplus V_{Adam}$$

$$\mathcal{A} = (V = V_{Eve} \uplus V_{Adam}, E)$$



Convention: Eve $\bigcirc$

Adam $\square$

Play

$$\pi = (v_0, v_1, v_2, \dots)$$

$$Play = Play_{Eve} \cup Play_{Adam}$$

$$Play_{Eve} = \{ (v_0, v_1, \dots, v_k) : \quad v_k \in V_{Eve} \}$$

$$Play_{Adam} = \{ \text{———} \quad V_{Adam} \}$$

Objectives

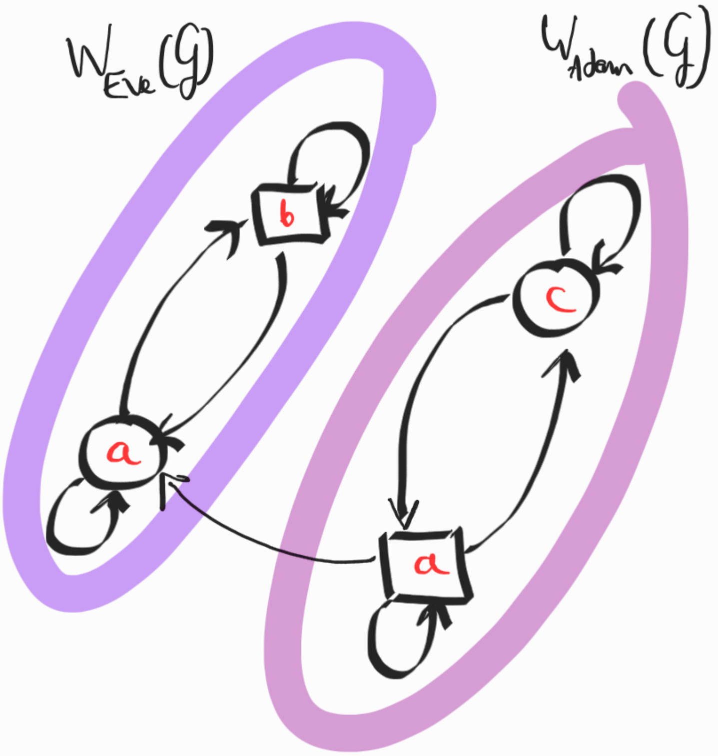
always: for Eve

$\Omega \subseteq C^\omega$ C colours

Games

$$\mathcal{G} = (\mathcal{A}, \Omega[col])$$

$col : V \longrightarrow C$



Example:

$$\Omega = \{\rho : \exists i \in \mathbb{N}, \rho_i = b\}$$

Strategies

$\sigma : \mathrm{Play}_{Eve} \to E$

$\tau : \mathrm{Play}_{Adam} \to E$

positional = memoryless : $\sigma : V_{Eve} \to E$

$\tau : V_{Adam} \to E$

Implicitely: $\sigma \rightsquigarrow$ Eve $\rightsquigarrow \bigcirc$

$\tau \rightsquigarrow$ Adam $\rightsquigarrow \square$

Plays consistent with a strategy:

$\pi = (v_1, v_2, \dots)$ is consistent with σ

$\forall i \in \mathbb{N} \quad v_i \in V_{Eve} \Rightarrow \sigma(v_1, \dots, v_i) = (v_i, v_{i+1}) \in E$

similar : π consistent with τ

Winning strategies

σ is winning from v if $\forall \pi$ consistent with σ starting in v

$$col(\pi) \in \Omega$$

similar : τ winning from v

Winning regions

$$W_{Eve}(\mathcal{G}) = \{v \in V : \exists \sigma \text{ winning from } v\}$$

Eve

$$W_{Adam}(\mathcal{G}) = \{v \in V : \exists \tau \text{ winning from } v\}$$

Adam

Determinacy: we say that $\mathcal{G}$ is determined if:

$$W_{Eve}(\mathcal{G}) \cup W_{Adam}(\mathcal{G}) = V$$

[$W_{Eve}(\mathcal{G}) \cap W_{Adam}(\mathcal{G}) = \emptyset$ is trivial and always holds]

Solving a game:

INPUT: $\mathcal{G}$ a game $v_0 \in V$

OUTPUT: Does Eve have a winning strategy from v_0 ?

$v_0 \in W_{Eve}(\mathcal{G})$?

n number of vertices

m number of edges