Exam MPRI

2019

Let G be a (two-player deterministic) mean-payoff game. The weights are integers and label edges. We let W denote the largest weight in absolute value. The mean-payoff objective is that the infimum limit of the weights seen along a path is non-negative ($\geqslant 0$).

Question 1: Prove that if Eve has a winning strategy, then she has a strategy that ensures that at all times, the total sum remains always larger than or equal to $-nW$. Is the value nW optimal?

Question 2: We construct a deterministic automaton A. The alphabet is the set of weights of G. The set of states is $[-nW, nW]$ plus an extra sink rejecting state $\perp$ and a sink accepting state $\top$. The automaton starts from the state 0 and stores the total sum of the weights, restricted to $[-nW, nW]$. The automaton rejects (goes to $\perp$) if the total sum goes below $-nW$. If the total sum goes above nW, it stays in $\top$. *Important*! **All** states except for $\perp$ are accepting.

Construct a safety game over the synchronised product of G and A such that Eve has a winning strategy in the mean-payoff game G if and only if Eve has a winning strategy in the safety game $G \times A$.

Question 3: Construct an algorithm for solving mean-payoff games based on the construction above. Analyse its (time) complexity.

Question 4: Construct an algorithm for solving CoBüchi games based on the construction above. Analyse its (time) complexity.

Question 5: Let k be the number of different weights in the mean-payoff game G. In the similar way as above, construct an algorithm for solving mean-payoff games whose complexity is $O(mn^k)$.

Question 6: In the constructions above, can we avoid constructing explicitly the automaton A to have a better space complexity for solving mean-payoff games?